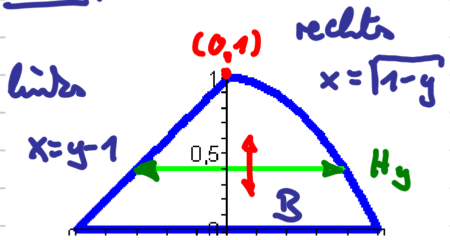
SS 2015 Analysis 4 LB Hausaufgaben Blatt 8

Notiztitel 06.05.2011

H13:

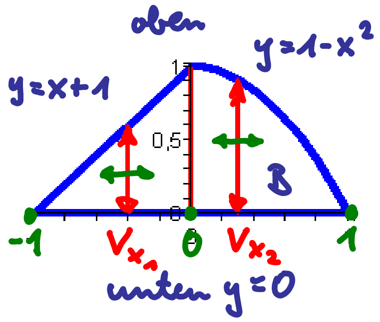


Darstellung von B durch Horizontalschnitte:

$$B=\bigcup_{0\le y\le 1} H_y = \{(x,y) \mid y-1\le x\le\sqrt{1-y},\ 0\le y\le 1\}$$

$$\Rightarrow F=\iint_B dF = \int_{y=0}^{1} \Big(\int_{x=y-1}^{\sqrt{1-y}} 1\,dx\Big)\,dy = \int_{y=0}^{1} \sqrt{1-y}-y+1\,dy$$

$$= -\tfrac23(1-y)^{3/2}-\tfrac12 y^2+y\Big|_{y=0}^{1} = -\tfrac12+1+\tfrac23=\underline{\underline{\tfrac76}}$$



Darstellung von B durch Vertikalschnitte:

$$B=\bigcup_{-1\le x\le 1} V_x = \{(x,y)\mid 0\le y\le 1+x,\ -1\le x\le 0\}\cup\{(x,y)\mid 0\le y\le 1-x^2,\ 0\le x\le 1\}$$

$$\Rightarrow F=\iint_B dF=\int_{x=-1}^{0}\Big(\int_{y=0}^{x+1}1\,dy\Big)\,dx+\int_{x=0}^{1}\Big(\int_{y=0}^{1-x^2}1\,dy\Big)\,dx=$$

$$=\int_{x=-1}^{0} x+1\,dx+\int_{x=0}^{1} 1-x^2\,dx = \big(\tfrac12x^2+x\big)\Big|_{x=-1}^{0}+\big(x-\tfrac13x^3\big)\Big|_{x=0}^{1} = -\tfrac12+1+1-\tfrac13=\underline{\underline{\tfrac76}}$$

Schwerpunktskoord. $S=(x_S,y_S)$: $x_S=\frac1F\iint_B x\,dy\,dx$; $y_S=\frac1F\iint_B y\,dy\,dx$

hier „zuerst nach y dann nach x":

$$x_S=\tfrac67\cdot\Big[\int_{x=-1}^{0}\Big(\int_{y=0}^{x+1}x\,dy\Big)dx+\int_{x=0}^{1}\Big(\int_{y=0}^{1-x^2}x\,dy\Big)dx\Big]=\tfrac67\cdot\Big[\int_{x=-1}^{0}xy\Big|_{y=0}^{x+1}dx+\int_{x=0}^{1}xy\Big|_{y=0}^{1-x^2}dx\Big]=$$

$$=\tfrac67\cdot\Big[\int_{x=-1}^{0}x^2+x\,dx+\int_{x=0}^{1}x-x^3\,dx\Big]=\tfrac67\Big[\big(\tfrac13x^3+\tfrac12x^2\big)\Big|_{x=-1}^{0}+\big(\tfrac12x^2-\tfrac14x^4\big)\Big|_{x=0}^{1}\Big]=$$

$$=\tfrac67\cdot\Big[\tfrac13-\tfrac12+\tfrac12-\tfrac14\Big]=\tfrac67\cdot\tfrac1{12}=\underline{\underline{\tfrac1{14}}}$$

$$y_S=\tfrac67\cdot\Big[\int_{x=-1}^{0}\Big(\int_{y=0}^{x+1}y\,dy\Big)dx+\int_{x=0}^{1}\Big(\int_{y=0}^{1-x^2}y\,dy\Big)dx\Big]=\tfrac67\cdot\Big[\int_{x=-1}^{0}\tfrac12y^2\Big|_{y=0}^{x+1}dx+\int_{x=0}^{1}\tfrac12y^2\Big|_{y=0}^{1-x^2}dx\Big]=$$

$$=\tfrac67\Big[\int_{x=-1}^{0}\tfrac12(x+1)^2\,dx+\int_{x=0}^{1}\tfrac12(1-x^2)^2\,dx\Big]=\tfrac37\Big[\big(\tfrac13x^3+x^2+x\big)\Big|_{x=-1}^{0}+\big(x-\tfrac23x^3+\tfrac15x^5\big)\Big|_{x=0}^{1}\Big]=$$

$$=\tfrac37\Big[\tfrac13-1+1+1-\tfrac23+\tfrac15\Big]=\tfrac37\cdot\tfrac{13}{15}=\underline{\underline{\tfrac{13}{35}}}$$

Berechne x_S und y_S auch mittels Integration zuerst über x dann über y